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BOOK REVIEW

Principles of Magnetohydrodynamics, with Applications to Laboratory and Astrophysical Plasma. By H. Goedbloed & S. Poedt. Cambridge University Press, 2004. 632 pp. ISBN 0521 623472, £80 (hardback); ISBN 0521 626072, £40 (paperback)

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Goedbloed & Poedt have written a unique and outstanding volume on theoretical magnetohydrodynamics (MHD). The narrative begins with the basic physical concepts, illustrated by a discussion of plasmas confined in a magnetic field in the laboratory and plasmas embedded in the vast magnetic fields in the astronomical universe. Chapter 1 emphasizes the Tokamak magnetic configuration and compares the conditions necessary for fusion in the plasma with the conditions in such places as the core and corona of the Sun, all with temperatures in the general range 0.1–10 Kev.

The theoretical dynamical properties of magnetic fields and plasmas are introduced with a review of the motion of an individual charged particle moving freely in a large-scale non-uniform magnetic field, including the adiabatic invariants of the motions, mirroring, etc. The velocity moments of the collisional Boltzmann equation are developed in the conventional manner to obtain the partial differential equations for conservation of particles and momentum density, followed by the energy equation and the coefficients of thermal conductivity, viscosity, and resistivity. In chapter 2 there is a section on collective phenomena and a development of plasma oscillations and Landau damping, before construction of the equations for the fluid (hydrodynamic) description of the plasma. The magnetic dissipation term is constructed using the scalar form of Ohm's law, and MHD follows with the tacit assumption that the system is not overwhelmed by resistive dissipation. And here it should be understood that the essential feature of MHD is that the magnetic field is carried bodily with the bulk motion of the plasma in the limit of vanishing resistivity.

Up to this point the text provides a substantial intellectual foundation for MHD, more or less along the usual lines. But then Chapter 3 launches into the elaborate formal derivation of the MHD equations from the kinetic approach: that a fluid is, after all, really just a collection of particles. The reader is carefully and clearly guided on a mathematical journey through the essential arguments, which serves as a concise road map across the vast territory of mathematical plasma kinetics (see T. Y. Wu, *Kinetic Equations of Gases and Plasma*, Addison Wesley, 1966 for a more detailed geography).

Chapter 4 breaks out into more familiar territory, redeveloping the basic equations of MHD from the fluid concept, as the authors promised at the beginning of Chapter 3. The authors refer to their fluid development as 'postulating' the basic equations, showing their dedication to rigorous mathematical derivation of the MHD equations from the basic laws of physics, namely particle dynamics, Maxwell's equations, and Lorentz transformations. This reviewer is more inclined to the view that the formal mathematical construction is instructive and desirable, but a judicious choice of the physical concepts leading to an obvious and elementary construction of the basic equations from first principles is on the same footing as the formal mathematical derivation, both being based on the empirical 'postulates' of Newton and Maxwell. Chapter 4 is quite clear on either basis. The authors go on to introduce the useful concept of the individual flux tube, based on the general smallness of the dissipation terms (resistivity, thermal conduction) worked out in the formal calculations of the preceding chapter. The MHD equations are subsequently cast in conservation form, in terms of entropy p/ρ^{γ}, thermal energy density, etc. with clear emphasis on the stresses in the magnetic field and their central role in the dynamical interplay with the pressure and inertia of the moving plasma.

It is gratifying to see these essential physical concepts of momentum, energy, and field stress emphasized in the MHD context because there are popular misconceptions today due to a turning away from the basic mechanics of magnetic stress working against the pressure and momentum of the plasma. The misconceptions have evolved into a counter-scientific culture in some fields of research, particularly magnetospheric physics, with an occasional attempt to inject the ideas into astrophysics. The misconception appears to have its genesis in the statement that magnetic fields embedded in a plasma are 'caused' by electric currents, in the same way that the magnetic field of a coil of wire in the laboratory is caused by the current flowing in the wire. From that it is concluded that the electric current is the fundamental field variable, rather than the magnetic field. This ignores the fact that the partial differential MHD equations, obtained directly from the equations of Newton and Maxwell, are expressed in terms of the magnetic field $\boldsymbol{B}$ and the bulk velocity $\boldsymbol{v}$ of the plasma. The magnetic field cannot be eliminated in favour of the current density $\boldsymbol{j}$ without using the Biot-Savart integral, thereby converting the tractable partial differential MHD equations to global integro-differential equations of so complicated a form as to be useless in the general case. So, lacking workable differential equations for describing the bulk dynamics of the field and plasma, the misconception turns to fantasy, taking the electric current density $\boldsymbol{j}$ to be related through the generalized Ohm's law to the electric field $\boldsymbol{E}$, and adopting $\boldsymbol{E}$ and $\boldsymbol{j}$ as the fundamental field variables rather than $\boldsymbol{B}$ and $\boldsymbol{v}$. Newton's equations of motion usually vanish from the picture. MHD is declared to be inapplicable to a collisionless or partially ionized plasma, and hence inapplicable almost everywhere in the magnetosphere. Marvellous proactive dynamical properties are imagined for the weak $O(v/c)$ electric field $\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}/c$, and it is often claimed that $\boldsymbol{j}$ is determined by the electric circuit equations so familiar in the laboratory. Indeed there are circumstances where an equivalent electric circuit can be constructed after the fact, but not in advance for a time-dependent dynamical system. One of the more outstanding predictions of this fanciful view is an enormous potential difference across a local blockage of the current, based on the belief that the circuit behaves dynamically as if the current were driven by the enormous equivalent inductance of the magnetic field. It overlooks the basic fact that there is no electric field in the frame of reference of the moving plasma. And, indeed, proper solution of the MHD equations shows that in fact there are no significant potential differences when a barrier is suddenly erected to block the current flow. The currents are instantly rerouted – something that does not happen in the laboratory circuit – and the system carries on in the manner of MHD. Not very exciting, perhaps, but consistent with Newton and Maxwell.

Unfortunately, many texts on MHD do not address the problem, directly constructing the fluid equations for MHD assuming the scalar Ohm's law, and going on from there to applications. The scalar Ohm's law has come to be viewed as an essential part of MHD theory. So the general development put forth by Goedbloed & Poedt, pursuing both the formal mathematical formulation from kinetic theory and the straightforward 'postulating' from the fluid concept, is a welcome addition to the existing literature. They do not go into the case of a partially ionized plasma, but the collisionless plasma is a limiting case of their general development.

Use of MHD is unavoidable for the large-scale bulk dynamics of a fluid containing a sufficient number of free electrons and ions. There are two necessary and sufficient conditions. First, the large-scale dynamics of any gas with a number density large enough to give an accurate statistical definition of the local gas density is described by the familiar hydrodynamic momentum equation, with the pressure tensor understood to represent the momentum flux density in the thermal motions. Second, if there are enough free electrons and ions that the sufficiently dense fluid cannot support any significant electric field in its own moving frame of reference, then the magnetic field is carried bodily with the fluid, and the large-scale bulk gas dynamics automatically becomes MHD. Interparticle collisions enter the picture only at the level of computing the pressure tensor and computing the dissipation arising from the non-zero electric field $\boldsymbol{E}'$ in the frame of the moving fluid. Both these tasks can be accomplished by elementary means in most cases (cf. E. N. Parker, *J. Geophys. Res.*, vol. 101, 1996, 10587).

Goedbloed & Poedt cast the appropriate MHD equations in one-fluid form and in two-fluid form, with the underlying kinetic theory clearly exhibited before going on to Part II of the book: Basic Magnetohydrodynamics. Laboratory plasma containment, discontinuities, waves, and characteristics are treated rigorously, turning to spectral theory where it is appropriate. The analogies with quantum mechanics are noted where appropriate. Stability and variational principles are treated with mathematical sophistication in both laboratory and astrophysical contexts, ending with resonant absorption and wave heating, which play such important roles in both laboratory plasma fusion and stellar coronas. In each case the level of mathematics is chosen to fit the problem, never shying from adequate mathematical power nor unnecessarily 'shooting a squirrel with a cannon'. The text carries the subject to an advanced level of development. So it is amusing to note on p. xiv of the preface that the authors remark that the topic of the forthcoming Volume II is to be *Advanced Magnetohydrodynamics*. Actually, they do admit to some advanced work in Volume I, with Volume II aimed then at nonlinear and non-ideal plasmas.

In summary, Volume I – the present volume – is an outstanding contribution to the subject of MHD theory and its applications. Supplemented with the implied Volume II, it might well become the definitive treatise on the subject. The work functions as both a general reference volume to the basic derivations for the practising plasma physicist and as an introductory textbook for those entering the field. The chapters are supported with extensive references to the general literature and with exercises involving the topics under discussion. The authors have marked the sections and paragraphs that might well be omitted on a first reading, guiding the student through the essentials in the long journey into theoretical plasma physics.

E. N. PARKER